THE INFLUENCE OF EARTH'S FLATTENING ON THE BEHAVIOR OF A HORIZONTAL GYROCOMPASS

(VLIIANIE SZHATIA ZEMLI NA RABOTU GIROGORIZONTKOMPASA)

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This paper presents investigations of a model of a force system proposed by Ishlinskii in [1-4] which acts on a material system moving close to earth's surface. Simultaneously with the new method Ishlinskii introduced new definitions of a local vertical and of a horizontal plane which differ from the conventional definitions. Consequently, the results presented in the above mentioned papers require special explanations.

In this paper the flattening of the earth is taken into account, the differential equations of motion of a horizontal gyrocompass are constructed and the conditions under which the instrument will show the true local vertical and the true meridional plane are determined.

1. We shall investigate the motion of a particle in the $Ox_1y_1z_1$ coordinate system which translates in an inertial reference frame while the point O moves arbitrarily on the earth's surface. The force $\mathbf{Q}$ which acts on a particle in the $Ox_1y_1z_1$ system can be represented by two equivalent expressions

$$\mathbf{Q} = \mathbf{P} - m\mathbf{w}^* \tag{1.1}$$

$$\mathbf{Q} = \mathbf{F} - m\mathbf{w} \tag{1.2}$$

Here m is the mass of the particle, $\mathbf{P} = m\mathbf{g}$ is the gravity force, which is the resultant of the earth's gravitational attraction force $\mathbf{F}$ and the centrifugal force $m\mathbf{w}_e$, where $\mathbf{w}_e$ is the translational acceleration of the point O caused by the earth's rotation. Thus

$$\mathbf{P} = \mathbf{F} - m\mathbf{w}_e \tag{1.3}$$

The vector $\mathbf{w}^*$ is the sum of two vectors $\mathbf{w}_r$ and $\mathbf{w}_c$, where $\mathbf{w}_r$ is the acceleration of the point O in its motion relative to earth's surface, and $\mathbf{w}_c$ is the Coriolis acceleration

$$\mathbf{w}^* = \mathbf{w}_r + \mathbf{w}_c \tag{1.4}$$

and $\mathbf{w}$ is the acceleration of the point O with respect to the inertial system

$$\mathbf{w} = \mathbf{w}_e + \mathbf{w}_r + \mathbf{w}_c \tag{1.5}$$

Direct measurements on earth's surface cannot determine either the magnitude or the direction of the gravitational attraction force $\mathbf{F}$. This direction does not coincide with the local vertical and the assumption that it is pointing toward the earth's center is only a first approximation [5-7]. The gravity force $\mathbf{P} = m\mathbf{g} = \mathbf{F} - m\mathbf{w}_e$ is being measured directly and the force $\mathbf{F}$ is not needed for its determination. The force $\mathbf{P}$ is directed along the local vertical and its projection on the horizontal plane equals zero. For these reasons, when due to conditions of a problem the motion must be referred to the local vertical and the horizontal plane, it is customary to use formula (1.1).

In the years 1956 and 1957 Ishlinskii proposed in [1-4] the use of formula (1.2) instead of (1.1); he assumed besides the sphericity of the earth and that the gravitational attraction force $\mathbf{F}$ is directed toward the earth's center along its radius. Under these assumptions Ishlinskii obtained many results on the equilibrium of a physical pendulum, on the behavior of complicated gyroscopic systems and on the systems of inertial navigation [1-4, 8]. In recent years a number of papers appeared by other authors, for example [9-14], who used similar assumptions.

In order to appreciate properly the new method and the results obtained from its application, it is necessary to examine certain ideas used by the authors of all these papers, without unduly stressing that the definitions in [1-4] differ from the conventional ones.

The local vertical, or the true vertical, is understood to be the line coinciding with the plumb line. The local vertical coincides also with the normal to the earth's surface (assuming that the earth is a geoid*). The horizontal plane is perpendicular to the local vertical.

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* In most books on theoretical mechanics it is assumed for simplicity that the earth is a sphere whereas the local vertical is always defined as coinciding both with the plumb line and the direction of the gravity force $\mathbf{P} = m\mathbf{g} = \mathbf{F} - m\mathbf{w}_e$. Without explaining the shape of the earth this inconsistency may lead to the wrong conclusion that the local vertical does not coincide with the normal to the earth's surface. The best explanation of this is in Sommerfeld [15].

The line between a point on the earth's surface and the earth's center will be called *pseudovertical* and the plane perpendicular to it *pseudohorizontal plane*. If the earth were a sphere then the pseudovertical would coincide with the normal to the earth's surface and the pseudohorizontal plane would coincide with the tangent plane. In Fig. 1 the geoid is drawn by a continuous ellipse-like curve, whereas the hypothetical spherical earth is drawn by the broken line; ζ is the true vertical, ζ' is the pseudovertical, η is the horizontal plane, η' is the pseudohorizontal plane.

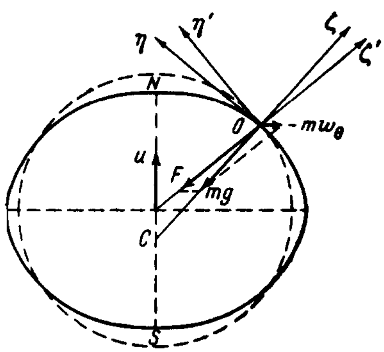

Fig. 1.

In the first approximation the angle ν between the vertical and the pseudovertical equals

$$\nu = \frac{1}{2}\frac{R_1 U^2}{g}\sin 2\varphi \qquad (1.6)$$

where U is the angular velocity of earth's rotation, R_1 is the principal radius of curvature represented by the segment OC (Fig. 1 and formulas (2.3)), g is the gravitational acceleration, φ is the geographical latitude.

It is assumed in [1-4,8] that the horizontal component of the gravity force **F** equals zero, therefore instead of the horizontal plane and the true vertical there are used in these papers the pseudohorizontal plane and the pseudovertical. Consequently, when all the requirements and conditions obtained in [2-4] for a gyrovertical and a horizontal gyrocompass are rigorously satisfied, these instruments should indicate not the horizontal but the pseudohorizontal plane. In [2], in particular, it is stated that the xy-plane which is fixed in the gyroframe remains horizontal during all manoeuvres of a ship, whereas in reality the word "horizontal" should be replaced by pseudohorizontal.

In the inertial navigation and in the theory of gyroscopic instruments the exact meaning of the horizontal plane and the local vertical concepts has fundamental importance for the following reasons: (1) If an accelerometer is in the pseudohorizontal plane and the base remains stationary relative to the earth, then it will show accelerations equalling $1/2\ R_1 U^2 \sin 2\varphi$ which are being determined from the horizontal component of the centrifugal force caused by the earth's rotation. (2) The angle ν can be as large as $6'$ which for many gyroscopic systems is prohibitively too large. (3) It is known that in the position of dynamic equilibrium the angle between the axis of a gyrocompass and the true horizontal plane β_r is not zero. The problem of a gyrocompass is to make the angle β_r vanish. In a gyrocompass with period equalling the M. Schuler period, the angle β_r on a stationary base is exactly given by the right-hand term in (1.6) (see, for example, [16]), which means that the axis of such a gyrocompass shows the pseudohorizontal plane. This occurs also in Ishlinskii's [2] horizontal gyrocompass, therefore these two instruments must in principle perform the same function and they may differ only in errors peculiar to their construction.

It is clear now why we must explain the accepted terminology and the results obtained when we use the assumptions of sphericity of the earth, of the direction of the gravitational force **F**, and chiefly of the expedience of expressing the force **Q** by formula (1.2). For this reason, in order to avoid possible misunderstandings and errors, which can arise from using such common concepts as the horizontal plane, the local vertical, the horizontal velocity component etc., we shall regard in the future the motion of a particle or of a system as being referred to the true vertical and true horizontal plane. We shall assume also, as is done in the classical textbooks on mechanics [15,17-19], that the force **Q** must be expressed by formula (1.1).

2. We shall consider first the problem of kinematics of a particle moving on the earth's surface. The earth is assumed to be a geoid on which normals to the surface coincide with the direction of the gravity force $\mathbf{P} = m\mathbf{g}$. The parameters which determine the dimensions and the shape of the geoid are

$$a = 6378.4 \text{ km}, \qquad \alpha = \frac{a-b}{a} = \frac{1}{296.3} \tag{2.1}$$

where a is the equatorial radius and b is the half distance between the poles.

To first order accuracy in α, the surface of the geoid can be approximated by the ellipsoid of Clairaut, where the eccentricity e of a meridian equals

$$e = \sqrt{2\alpha - \alpha^2} \tag{2.2}$$

From now on we shall regard the surface of the earth as being a Clairaut ellipsoid and in all expansion in powers of the small parameter α (or e^2) we shall keep only first order terms.

The principal normal sections of an ellipsoid of revolution through the point O are the plane of the first vertical and the meridional plane. The corresponding principal radii of curvature are given by the equations

$$R_1 = \frac{a}{\sqrt{1 - e^2 \sin^2\varphi}}, \qquad R_2 = \frac{a(1-e^2)}{(1 - e^2 \sin^2\varphi)^{3/2}} \tag{2.3}$$

where φ is the geographical latitude. The first principal radius of curvature (Fig. 1) is represented by the segment OC and the second one is the radius of curvature of the meridian. Let us mention the easily derived and useful identity

$$\frac{\partial (R_1 \cos\varphi)}{\partial\varphi} = -R_2 \sin\varphi \tag{2.4}$$

We shall introduce now the geographically oriented coordinate system $O\xi\eta\zeta$ (the ζ-axis is along the true vertical pointing up, the ξ and η axes are horizontal, the ξ-axis is directed east and the η-axis directed north), and denote by v_E, v_N, v_ζ the ξ, η, ζ components of the velocity of the point O with respect to the earth. Since the point O moves on the surface of the earth, the components of the velocity vector $\mathbf{V}$ of the point O with respect to the inertial frame of [16] are

$$V_\xi = v_E + R_1 U \cos\varphi, \quad V_\eta = v_N, \quad V_\zeta = 0 \tag{2.5}$$

The time rates of the latitude φ and of the longitude λ equal

$$\dot\varphi = \frac{v_N}{R_2}, \qquad \dot\lambda = \frac{v_E}{R_1\cos\varphi} \tag{2.6}$$

The angular velocity of the triad $\xi\eta\zeta$ is given by the equations

$$u_\xi = -\frac{V_\eta}{R_2}, \qquad u_\eta = \frac{V_\xi}{R_1}, \qquad u_\zeta = \frac{V_\xi}{R_1}\tan\varphi \tag{2.7}$$

The simplest way to calculate the components of the velocity vector of the point O with respect to the inertial reference frame [18] is by using the formulas

$$w_\xi = \dot V_\xi + u_\eta V_\zeta - u_\zeta V_\eta, \qquad w_\eta = \dot V_\eta + u_\zeta V_\xi - u_\xi V_\zeta$$
$$w_\zeta = \dot V_\zeta + u_\xi V_\eta - u_\eta V_\xi$$

Substituting (2.7) in the above equations we obtain

$$w_\xi = \dot V_\xi - \frac{V_\xi V_\eta}{R_1}\tan\varphi, \quad w_\eta = \dot V_\eta + \frac{V_\xi^2}{R_1}\tan\varphi, \quad w_\zeta = -\left(\frac{V_\xi^2}{R_1} + \frac{V_\eta^2}{R_2}\right) \tag{2.8}$$

If we used the equations (2.4) and (2.5) we would obtain

$$\begin{aligned} w_\xi &= \dot v_E - \frac{v_E v_N}{R_1}\tan\varphi - 2U v_N \sin\varphi \\ w_\eta &= \dot v_N + \frac{v_E^2}{R_1}\tan\varphi + 2U v_E \sin\varphi + U^2 R_1 \sin\varphi\cos\varphi \\ w_\zeta &= -\frac{v_E^2}{R_1} - \frac{v_N^2}{R_2} - 2U v_E \cos\varphi - U^2 R_1 \cos^2\varphi \end{aligned} \tag{2.9}$$

In the above expressions the terms containing the angular velocity of the earth's rotation in the second degree are components of the transport acceleration $\mathbf{w}_e$ which is caused by the earth's rotation; the terms containing U in the first degree are components of the Coriolis acceleration $\mathbf{w}_c$; finally the terms which do not contain U are components of the acceleration $\mathbf{w}_r$ of the point O in its motion relative to the earth. Thus the ξ, η, ζ components of the vector $\mathbf{w}^*$ determined by (1.4) equal

$$\begin{aligned} w_\xi^* &= \dot{v}_E - \frac{v_E v_N}{R_1}\tan\varphi - 2Uv_N\sin\varphi \\ w_\eta^* &= \dot{v}_N + \frac{v_E^2}{R_1}\tan\varphi + 2Uv_E\sin\varphi \\ w_\zeta^* &= -\frac{v_E^2}{R_1} - \frac{v_N^2}{R_2} - 2Uv_E\cos\varphi \end{aligned} \tag{2.10}$$

Let us denote by V_ξ^{*2} and V^{*2}, respectively the approximate values of V_ξ^2 and V^2 when the terms containing squares of the angular velocity of the earth's rotation are neglected

$$V_\xi^{*2} = v_E^2 + 2R_1 v_E U\cos\varphi, \qquad V^{*2} = V_\xi^{*2} + V_\eta^2 \tag{2.11}$$

and let us introduce the small, variable parameter

$$\mu = e^2\cos^2\varphi \tag{2.12}$$

Then, with higher order accuracy in μ than the first, we have

$$\begin{aligned} w_\xi^* = \dot{V}_\xi - \frac{V_\xi V_\eta}{R_1}\tan\varphi, \quad w_\eta^* = \dot{V}_\eta + \frac{V_\xi^{*2}}{R_1}\tan\varphi \\ w_\zeta^* = -\frac{V^{*2}}{R_1}\left(1 + \mu\frac{V_\eta^2}{V^{*2}}\right) \end{aligned} \tag{2.13}$$

We shall write down the following two formulas (valid within the specified accuracy)

$$\frac{R_1}{R_2} = 1 + \mu, \qquad \dot{\mu} = -2\mu\frac{V_\eta}{R_1}\tan\varphi \tag{2.14}$$

Let us mention that all the formulas which we have already obtained and those which will follow can be easily generalized for cases when the point O has vertical displacements.

3. In the theory of gyroscopic instruments we often use a coordinate system in which the x^0y^0-plane is horizontal and the y^0-axis coincides with the horizontal component of the angular velocity of the triad $\xi\eta\zeta$ (Fig. 2). By (2.7) the angle ϑ is determined through

$$\tan\vartheta = \frac{R_1}{R_2}\frac{V_\eta}{V_\xi} \tag{3.1}$$

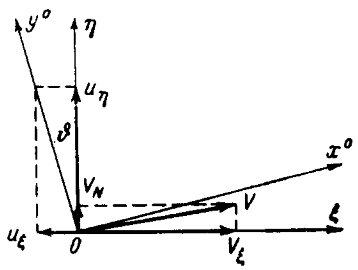

Fig. 2.

Within the specified accuracy we obtain

$$\tan\vartheta = (1+\mu)\frac{V_\eta}{V_\xi}, \quad \sin\vartheta = \frac{V_\eta}{V}\left(1+\mu\frac{V_\xi^2}{V^2}\right)$$

$$\cos\vartheta = \frac{V_\xi}{V}\left(1-\mu\frac{V_\eta^2}{V^2}\right) \tag{3.2}$$

Let us now derive formulas for the x°, y°, z° components of the velocity $\mathbf{V}$

$$V_{x^\circ} = V_\xi \cos\vartheta + V_\eta \sin\vartheta, \quad V_{y^\circ} = -V_\xi \sin\vartheta + V_\eta \cos\vartheta, \quad V_{z^\circ} = V_\zeta$$

With higher order accuracy than the first we find

$$V_{x^\circ} = V, \qquad V_{y^\circ} = -\mu\frac{V_\xi V_\eta}{V}, \qquad V_{z^\circ} = 0 \tag{3.3}$$

(it is useful to mention that the velocity vector $\mathbf{V}$ does not coincide with the x°-axis, as it does in the case of the spherical earth).

Similarly, we obtain the x°, y° and z° components of the acceleration $\mathbf{w}^*$

$$\begin{aligned} w_{x^\circ}^* &= \dot V - \frac{V_\eta}{R_1 V}\left(V_\xi^2 - V_\xi^{*2}\right)\tan\varphi + \mu\frac{V_\xi V_\eta}{V}\left(\dot\vartheta_0 + \frac{V_\xi}{R_1}\frac{V^{*2}}{V^2}\tan\varphi\right) \\ w_{y^\circ}^* &= \left(\dot\vartheta_0 + \frac{V_\xi}{R_1}\frac{V^{*2}}{V^2}\tan\varphi\right)V - \mu\frac{V_\xi V_\eta}{V^2}\left[\dot V - \frac{V_\eta}{R_1 V}\left(V_\xi^2 - V_\xi^{*2}\right)\tan\varphi\right] \\ w_{z^\circ}^* &= -\frac{V^{*2}}{R_1}\left(1+\mu\frac{V_\eta^2}{V^{*2}}\right) \end{aligned} \tag{3.4}$$

In these formulas $\dot\vartheta_0$ is the time derivative of the angle ϑ obtained as if the earth's flattening were absent

$$\dot\vartheta_0 = \frac{V_\xi \dot V_\eta - \dot V_\xi V_\eta}{V^2} \tag{3.5}$$

The x°, y° and z° components of the angular velocity ω_e of the rotation of the triad $x^\circ y^\circ z^\circ$ with respect to the inertial frame of reference have the form

$$\omega_{ex^\circ} = 0, \qquad \omega_{ey^\circ} = \omega_1, \qquad \omega_{ez^\circ} = \omega \tag{3.6}$$

where

$$\omega_1 = \frac{V}{R_1}\left(1+\mu\frac{V_\eta^2}{V^2}\right), \qquad \omega = \omega_0 + \mu\left(\dot\vartheta_0 - 2\frac{V_\eta^2}{V^2}\omega_0\right) \tag{3.7}$$

Here ω_0 is the value of ω when $\mu = 0$

$$\omega_0 = \dot\vartheta_0 + \frac{V_\xi}{R_1}\tan\varphi \tag{3.8}$$

The parameter μ is small when compared to unity and varies within the limits $0 \leqslant \mu \leqslant 0.00675$, with $\mu = 0$ on a pole. For this reason in some cases the x°, y° and z° components of the acceleration $\mathbf{w}^*$ can be calculated sufficiently accurately for practical purposes by the formulas

$$w_{x^\circ}{}^* = \dot{V} - \frac{V_\eta}{R_1 V}(V_\xi{}^2 - V_\xi{}^{*2}) \tan \varphi$$
$$w_{y^\circ}{}^* = \left(\dot{\vartheta}_0 + \frac{V_\xi}{R_1}\frac{V^{*2}}{V^2}\tan\varphi\right)V, \qquad w_{z^\circ}{}^* = -\frac{V^{*2}}{R_1} \tag{3.9}$$

Finally, if in the above formulas we delete the superscript *, we obtain the x°, y° and z° components of the total acceleration of the point moving on a spherical earth's surface [2]

$$w_{x^\circ} = \dot{V}, \qquad w_{y^\circ} = \omega_0 V, \qquad w_{z^\circ} = -\frac{V^2}{R} \tag{3.10}$$

4. We shall calculate now the x°, y° and z° components of the force $\mathbf{Q}$ determined previously by (1.1). Since the force $\mathbf{P} = m\mathbf{g}$ is directed along a normal to the earth's surface we have

$$Q_{x^\circ} = -mw_{x^\circ}{}^*, \qquad Q_{y^\circ} = -mw_{y^\circ}{}^*, \qquad Q_{z^\circ} = -m(g + w_{z^\circ}{}^*) \tag{4.1}$$

where $w_{x^\circ}{}^*$, $w_{y^\circ}{}^*$ and $w_{z^\circ}{}^*$ should be calculated by formulas (3.4).

If the force $\mathbf{Q}$ is obtained from the equation (1.2) as in [1-4,8] then its x°, y° and z° components equal

$$Q_{x^\circ}{}' = -mw_{x^\circ} = -m\dot{V}, \quad Q_{y^\circ}{}' = -mw_{y^\circ} = -m\omega_0 V$$
$$Q_{z^\circ}{}' = -(F + mw_{z^\circ}) = -\left(F - m\frac{V^2}{R}\right) \tag{4.2}$$

(in this case the z°-axis is along the pseudovertical, and the x° and y° axes are in the pseudohorizontal plane).

In order to avoid mixing up the components (4.1) with the components (4.2) the latter are marked by primes.

It can be easily shown that the horizontal components Q_{x° and Q_{y° of the force $\mathbf{Q}$ may differ substantially from its pseudohorizontal components $Q_{x^\circ}{}'$ and $Q_{y^\circ}{}'$. For example when $v_E = 0$ and $v_N = \text{const}$ we have $Q_{x^\circ} : Q_{x^\circ}{}' \approx 2$. The components Q_{z° and $Q_{z^\circ}{}'$, however, differ very little because $F \approx mg$ and at relatively small velocities (say $v \leqslant 50$ knots)

$$\frac{v^2}{R_1} < \frac{V^2}{R_1} \ll g$$

5. We shall derive the differential equations of the precessional motion of a gyroframe using the results obtained by Ishlinskii in [2]. To construct these equations while relating the motion to the true horizontal plane, we can replace in the differential equations (40) of [2] the components of the acceleration $\mathbf{w}$ which are given in (3.10) by the components of $\mathbf{w}^*$ which are given by (3.4); besides, the two components V/R and ω_0 of the angular velocity ω_e of the triad $Ox^0y^0z^0$ with respect to the inertial frame of reference must be replaced by the corresponding expressions ω_1 and ω as given by (3.7) and (3.8). After performing the indicated substitutions we obtain the desired differential equations

$$\begin{gathered}
2B\cos\varepsilon\,[(\dot\alpha+\omega)\cos\beta\cos\gamma+\dot\beta\sin\gamma+\omega_1(\sin\alpha\sin\gamma-\cos\alpha\sin\beta\cos\gamma)]=\\
=ml\,[-w_{x^\circ}{}^*\sin\alpha\,\cos\beta+w_{y^\circ}{}^*\cos\alpha\cos\beta+(g+w_{z^\circ}{}^*)\sin\beta]-M_x{}'\\
\frac{d}{dt}(2B\cos\varepsilon)=ml\,[w_{x^\circ}{}^*(\cos\alpha\cos\gamma-\sin\alpha\sin\beta\sin\gamma)+\\
+w_{y^\circ}{}^*(\sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma)-(g+w_{z^\circ}{}^*)\cos\beta\sin\gamma]+M_y{}'\\
2B\cos\varepsilon\,[-(\dot\alpha+\omega)\cos\beta\sin\gamma+\dot\beta\cos\gamma+\omega_1(\sin\alpha\cos\gamma+\cos\alpha\sin\beta\sin\gamma)]=M_z{}'\\
2B\sin\varepsilon\,[(\dot\alpha+\omega)\sin\beta+\dot\gamma+\omega_1\cos\alpha\cos\beta]=-N
\end{gathered}\tag{5.1}$$

In these equations $M_x{}'$, $M_y{}'$ and $M_z{}'$ are supplementary moments which can be generated by a special arrangement; the remaining symbols have the same meaning as in [2], while the angles α, β, γ differ from the corresponding angles in [2] because they are defined in the system $x^0y^0z^0$ where the z^0-axis does not coincide with the true vertical but with the pseudovertical.

In order to make our gyroframe behave like a gyrocompass it is necessary to select such values for the moments N, $M_x{}'$, $M_y{}'$ and $M_z{}'$ which would permit the motion $\alpha = \beta = \gamma = 0$. Substituting $\alpha = \beta = \gamma = 0$ in (5.1) we obtain

$$\begin{aligned}
&2B\omega\cos\varepsilon=mlw_{y^\circ}{}^*-M_x{}', && 0=M_z{}'\\
&\frac{d}{dt}(2B\cos\varepsilon)=mlw_{x^\circ}{}^*+M_y{}', && 2B\omega_1\sin\varepsilon=-N
\end{aligned}\tag{5.2}$$

From where

$$2B\cos\varepsilon=\frac{mlw_{y^\circ}{}^*-M_x{}'}{\omega},\qquad M_y{}'=\frac{d}{dt}\left[\frac{mlw_{y^\circ}{}^*-M_x{}'}{\omega}\right]-mlw_{x^\circ}{}^*\qquad N=-2B\omega_1\sin\varepsilon\tag{5.3}$$

The moment $M_x{}'$ is arbitrary, besides, if $M_x{}' = M_x{}'(\varepsilon)$ then N can also be regarded as a function of the angle ε because the time t and the angle ε are related as shown in the first equation of (5.3).

If the conditions (5.3) are satisfied while the motion is taking place and at the initial instant $\alpha = \beta = \gamma = 0$, then the gyroframe will be in equilibrium with respect to the system $x^O y^O z^O$, showing always the true local vertical and the meridional plane (accurate within the course correction equalling ϑ).

From the equations (5.3) it is easy to obtain the Ishlinskii conditions. To achieve this we set $M_x' = 0$ and replace the components $w_{x^O}^*$ and $w_{y^O}^*$ by $w_{x^O} = \dot{V}$ and $w_{y^O} = \omega_0 V$; besides, we must take into account that in the case of a sphere $\omega_1 = V/R$ and $\omega = \omega_0$ (formulas (3.7) and (3.10)). Substituting these expressions in the conditions (5.3) we find (compare with [2])

$$M_y' \equiv 0, \qquad 2B\cos\varepsilon' = mlV, \qquad N = -\frac{4B^2 \sin\varepsilon' \cos\varepsilon'}{Rml} \tag{5.4}$$

Let us repeat that replacing the components of the acceleration $\mathbf{w}^*$ by the components of the total acceleration $\mathbf{w}$ means replacing the true vertical by the pseudovertical. Therefore under the conditions (5.4) the gyroframe will not be showing the horizontal plane (in order not to mix up the angles α, β, γ, ε with the corresponding angles in [2] we have written the angle ε in (5.4) with a prime).

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